Recursive Orthogonal Bisection

Marcus Rickert

email: mr@zpr.uni-koeln.de

Zentrum für Paralleles Rechnen
Universität zu Köln

Center for Parallel Computing
University of Cologne and
TSASA - Los Alamos National Lab

Graph Partitioning

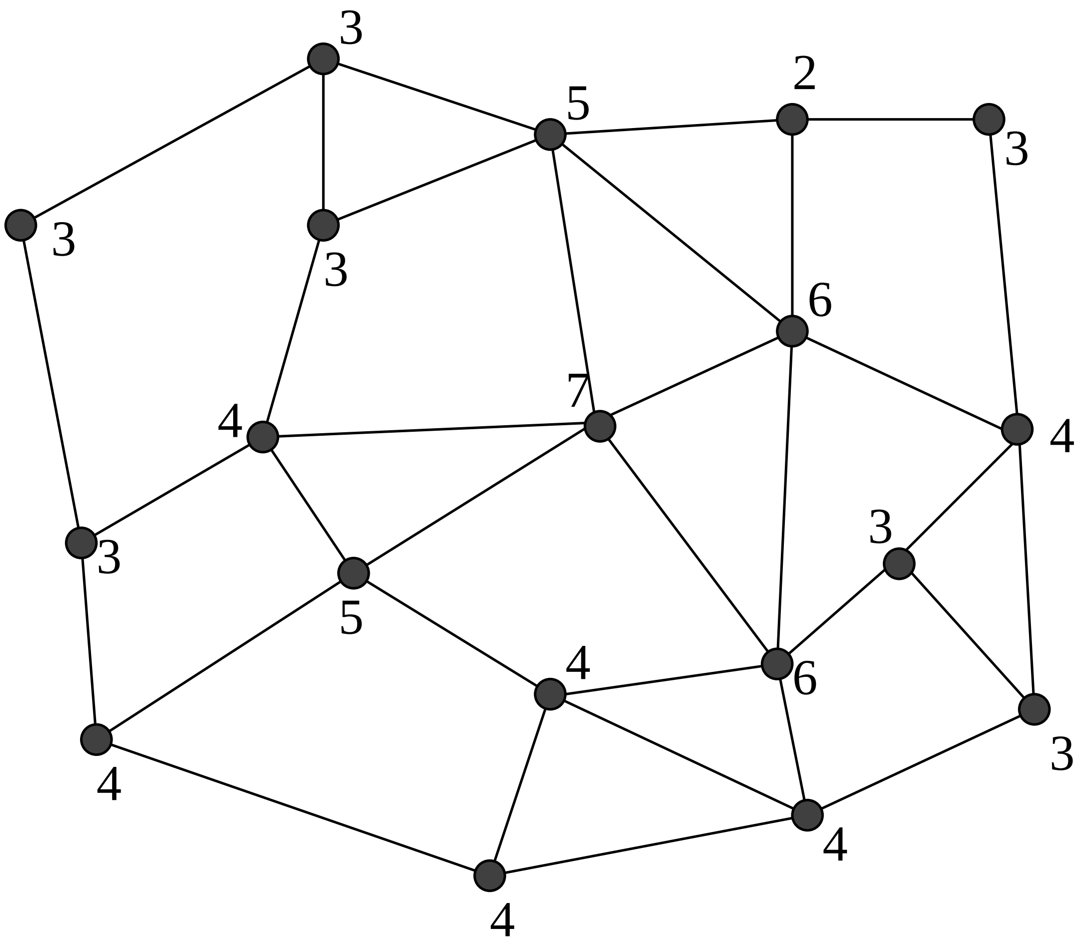
How do we distribute
N vertices $(v_1 \ldots v_N)$
of a graph $G = (V, E)$
onto P CPN while

- keeping communication costs as low as possible, and
- keeping load imbalance as small as possible.

The distribution is described by:

$$p : \begin{array}{ccc} V & \rightarrow & P \\ v_i & \mapsto & CPU_{p(i)} \end{array}$$

Geometry



Graph G(V, E)

$\bullet\, l_i$ vertex i and its load

Overall Communication Costs

Costs per edge (i, j)

$$c_{ij} = \begin{cases} c & \text{if } (i,j) \in E \wedge p(i) \neq p(j) \\ 0 & \text{otherwise} \end{cases}$$

Sum costs

$$C_{comm}(p) = \sum_{i,j} c_{ij}(p)$$

Assumption: costs to communicate between CPN are independent of their distance in CPN network

Local communication costs

Communication costs per CPN

$$C_k = \sum_{p(i)=k \vee p(j)=k} c_{ij}$$

Worst-case local communication

$$C_{max} = \max_{j \in P} C_j$$

Load Imbalance Costs

Load on CPN j

$$L(j) = \sum_{p(i)=j} l(i)$$

Minimum and maximum load

$$\begin{aligned} L_{min} &= \min_{j \in P} L(j) \\ L_{max} &= \max_{j \in P} L(j) \end{aligned}$$

Load imbalance:

$$C_{load} = L_{max} - L_{min}$$

Overall costs

Bus communication network

$$C_{comm}^{eff} \simeq C_{comm}$$

2 dimensional communication network

$$C_{comm}^{eff} \simeq C_{max}$$

Use t_L and t_C to convert to time.

Mininize:

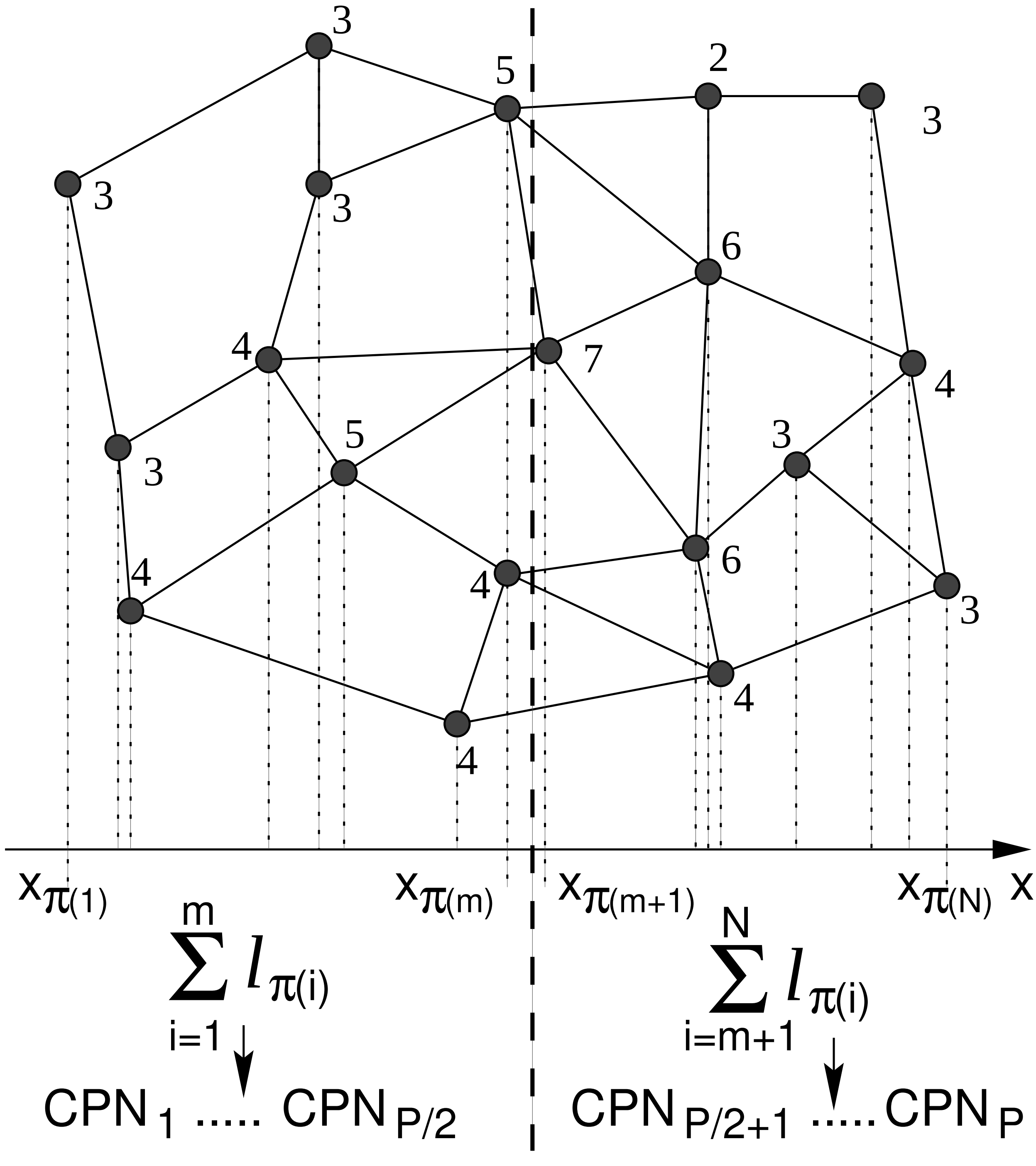
$$C = t_L C_{load} + t_C C_{comm}^{eff}$$

Recursive Orthognal Bisection

In a planar and homogenenous graph: recursive split graph along vertical and horizontal lines, while minimizing load-imbalance:

1. Sort vertices in set A by X (Y)
2. Split vertices vertically (horizontally) so that load imbalance between subsets A_1 and A_2 is minimal
3. Change orientation and recursively call routine for A_1 and A_2

Vertical Bisection



Horizontal Bisection